(61) Evaluate $\underset{x\to 0}{Lt}\left(\frac{\tan x}{x}\right)^{1/x}$

Ans. $\to$ Let $y = \underset{x\to 0}{Lt}\left(\frac{\tan x}{x}\right)^{1/x}$ $[1^{\infty}]$

Taking log boths sides, we have

$$\log_e y = \underset{x\to 0}{Lt}\ \frac{1}{x}\cdot\log\left(\frac{\tan x}{x}\right)$$

$$= \underset{x\to 0}{Lt}\ \frac{\log\left(\frac{\tan x}{x}\right)}{x}\quad \left[\frac{0}{0}\right]$$

$$= \underset{x\to 0}{Lt}\ \frac{\log\tan x - \log x}{x}$$

$$= \underset{x\to 0}{Lt}\ \frac{\frac{1}{\tan x}\cdot\sec^2 x - \frac{1}{x}}{}$$

$$= \underset{x\to 0}{Lt}\ \frac{\frac{1}{\cos^2 x}}{\frac{\sin x}{\cos x}} - \frac{1}{x} = \underset{x\to 0}{Lt}\ \frac{1}{\cos^{\cancel{2}} x}\times\frac{\cancel{\cos x}}{\sin x} - \frac{1}{x}$$

$$= \underset{x\to 0}{Lt}\ \frac{2}{2\sin x\cdot\cos x} - \frac{1}{x}$$

$$\log_e y = \lim_{x\to 0} \frac{2}{\sin 2x} - \frac{1}{x}$$

$$= \lim_{x\to 0} \frac{2x - \sin 2x}{x \sin 2x} \left[\frac{0}{0}\right]$$

$$= \lim_{x\to 0} \frac{2 - 2\times\cos 2x}{x \cdot 2\cos 2x + \sin 2x \times 1} \left[\frac{0}{0}\right]$$

$$= \lim_{x\to 0} \frac{-2\times - \sin 2x \times 2}{2\left[x\times - \sin 2x \times 2 + \cos 2x \times 1\right] + \cos 2x \times 2}$$

$$= \frac{0}{2[0+1] + 2\times 1} = \frac{0}{4} = 0$$

$\therefore \log_e y = 0$

$\therefore y = e^0 = 1$ Ans.

(62) Evaluate $\lim_{x\to 0} \left(\frac{\tan x}{x}\right)^{1/x^2}$.

Ans. → Let $y = \lim_{x\to 0} \left(\frac{\tan x}{x}\right)^{1/x^2}$ $[1^\infty]$

Taking log boths sides, we have,

$$\log_e y = \lim_{x\to 0} \frac{1}{x^2} \cdot \log\left(\frac{\tan x}{x}\right)$$

$$= \lim_{x\to 0} \frac{\log\left(\frac{\tan x}{x}\right)}{x^2} \left[\frac{0}{0}\right]$$

$$\log_e y = \underset{x\to 0}{Lt} \frac{\log\tan x - \log x}{x^2}$$

$$= \underset{x\to 0}{Lt} \frac{\frac{1}{\tan x}\cdot \sec^2 x - \frac{1}{x}}{2x}$$

$$= \underset{x\to 0}{Lt} \frac{\frac{\sec^2 x}{\tan x} - \frac{1}{x}}{2x} = \underset{x\to 0}{Lt} \frac{\frac{\frac{1}{\cos^2 x}}{\frac{\sin x}{\cos x}} - \frac{1}{x}}{2x}$$

$$= \underset{x\to 0}{Lt} \frac{\frac{1}{\cos x\cdot\sin x} - \frac{1}{x}}{2x}$$

$$= \underset{x\to 0}{Lt} \frac{\frac{2}{2\cos x\cdot\sin x} - \frac{1}{x}}{2x}$$

$$= \underset{x\to 0}{Lt} \frac{\frac{2}{\sin 2x} - \frac{1}{x}}{2x}$$

$$= \underset{x\to 0}{Lt} \frac{\frac{2x - \sin 2x}{x\sin 2x}}{2x} = \underset{x\to 0}{Lt} \frac{2x - \sin 2x}{x\sin 2x} \times \frac{1}{2x}$$

$$= \underset{x\to 0}{Lt} \frac{2x - \sin 2x}{2x^2\sin 2x} \quad \left[\frac{0}{0}\right]$$

$$= \underset{x\to 0}{Lt} \frac{2 - 2\cos 2x}{2\cdot 2x\sin 2x + 4x^2\cos 2x} \quad \left[\frac{0}{0}\right]$$

$$= \underset{x\to 0}{Lt} \frac{+2\times + \sin 2x\times 2}{4\sin 2x + 8x\cos 2x - 8x^2\sin 2x} \quad \left[\frac{0}{0}\right]$$

$$\log_e y = \lim_{x\to 0} \frac{4\sin 2x}{4\sin 2x + 8x\cos 2x + 8x\cos 2x - 8x^2\sin 2x} \quad \left[\frac{0}{0}\right]$$

$$\log_e y = \lim_{x\to 0} \frac{4\times \cos 2x\times 2}{4\times\cos 2x\times 2 + 8\times 1\cdot\cos 2x\times 2 + 8x\sin 2x\times 2\times 1 - 8\times 2x\sin 2x - 8x^2\cos 2x\times 2\times 1}$$

$\cancel{16\cos 2x}$

$$\log_e y = \lim_{x\to 0} \frac{8\cos 2x}{8\cos 2x + 16\cos 2x - 32x\sin 2x - 16x\sin 2x - 16x^2\cos 2x}$$

$$= \lim_{x\to 0} \frac{8\times 2}{16+32-0-0-0}$$

$$= \frac{\cancel{16}\,\cancel{8}}{\cancel{48}\,\cancel{24}\,3} = \frac{1}{3}$$

$\therefore \log_e y = \frac{1}{3}$

$\therefore y = e^{\frac{1}{3}}$ Ans.

(63) Evaluate $\lim_{x\to 0} \log_{\tan x} \tan 2x$.

Ans. $\to \lim_{x\to 0} \log_{\tan x} \tan 2x$

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$$= \lim_{x\to 0} \log_e \tan 2x \times \log_{\tan x} e$$

$$= \lim_{x\to 0} \frac{\log_e \tan 2x}{\log_e \tan x} \quad \left[\frac{\infty}{\infty}\right]$$

Hence from L' Hospital's Rule, we have

$$= \underset{x\to 0}{Lt} \frac{\frac{1}{\tan 2x} \times \sec^2 2x \times 2}{\frac{1}{\tan x} \times \sec^2 x \times 1}$$

$$= \underset{x\to 0}{Lt} \frac{2 \times \frac{\frac{1}{\cancel{\cos^2 2x}\cos 2x}}{\frac{\sin 2x}{\cancel{\cos 2x}}}}{\frac{\frac{1}{\cancel{\cos^2 x}\cos x}}{\frac{\sin x}{\cancel{\cos x}}}}$$

$$= \underset{x\to 0}{Lt} \frac{2 \times \frac{1}{\cos 2x} \times \frac{1}{\sin 2x}}{\frac{1}{\cos x \cdot \sin x}}$$

$$= \underset{x\to 0}{Lt}\ 2 \frac{1}{\cos 2x \cdot \sin x} \times \frac{\cos x \cdot \sin x}{1}$$

$$\cancel{= \underset{x\to 0}{Lt} \frac{2 \sin x \cdot \cos x}{\sin 2x \cdot \cos 2x}} = \underset{x\to 0}{Lt}\ 2 \cdot \frac{\cos x \cdot \sin x}{\cos 2x \cdot \sin 2x}$$

$$\cancel{= \underset{x\to 0}{Lt} \frac{\sin 2x}{\sin 2x \cdot \cos 2x} = \underset{x\to 0}{Lt} \frac{1}{\cos 2x} = \frac{1}{0}}$$

$$\cancel{= \frac{1}{1} = 1 \text{ Ans.}} = \underset{x\to 0}{Lt} \frac{\sin 2x}{\sin 4x} = \underset{x\to 0}{Lt} \frac{2 \cos 2x}{4 \cos 4x}$$

$$= \frac{1}{1} = 1 \text{ Ans.}$$